MAT8034: Machine Learning

Reinforcement Learning II

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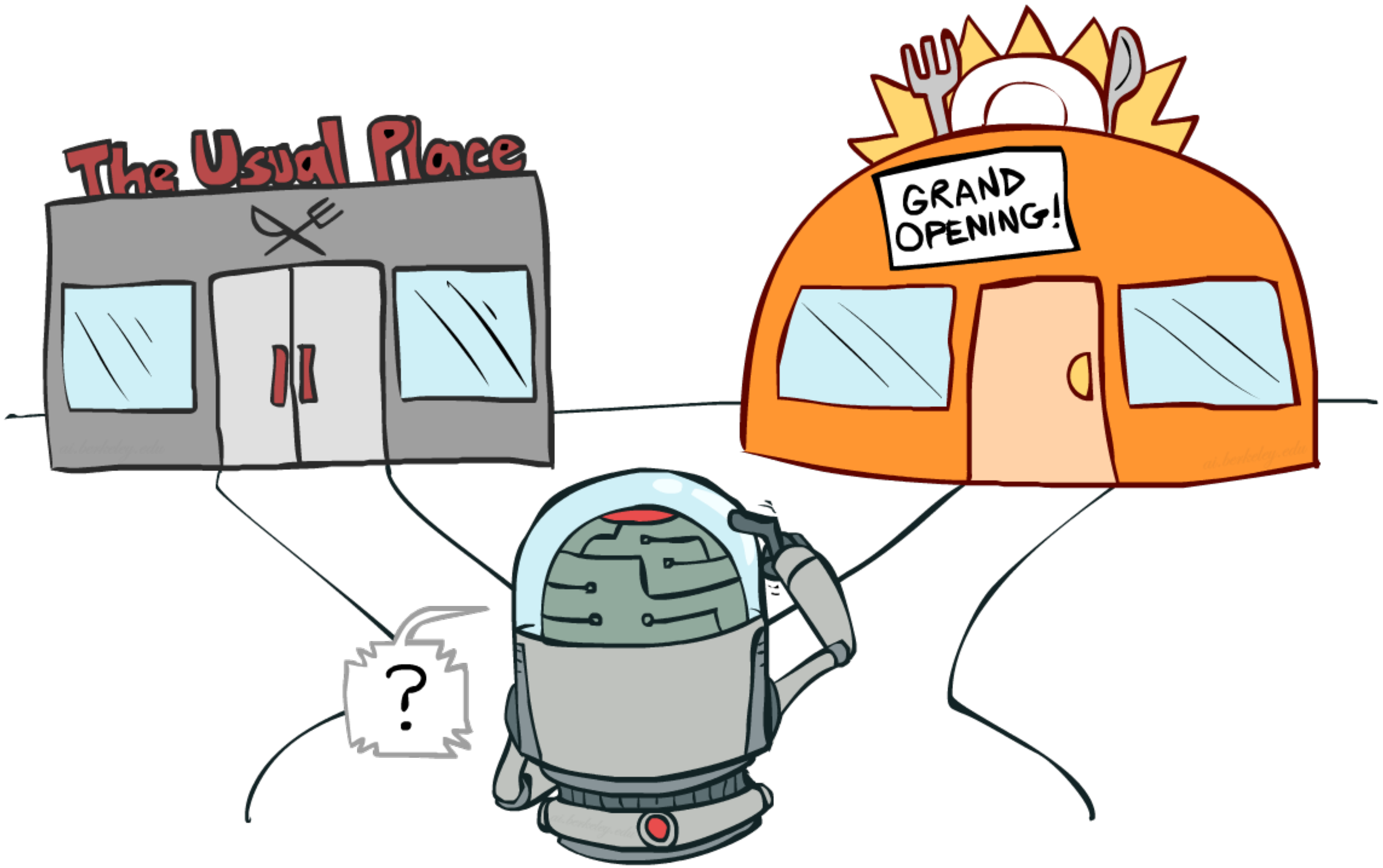
https://fangkongx.github.io/Teaching/MAT8034/Spring2025/index.html

Slide credits: ai.berkeley.edu

Exploration vs. Exploitation



Exploration vs. Exploitation

- ***Exploration***: try new things
- ***Exploitation***: do what's best given what you've learned so far
- Key point: pure exploitation often gets ***stuck in a rut*** and never finds an optimal policy!

Exploration method 1: ε-greedy

- ε-greedy exploration
  - Every time step, flip a biased coin
  - With (small) probability ε, act randomly
  - With (large) probability 1-ε, act on current policy
- Properties of ε-greedy exploration
  - Every s,a pair is tried infinitely often
  - Does a lot of stupid things
    - Jumping off a cliff *lots of times* to make sure it hurts
  - Keeps doing stupid things for ever
    - Decay ε towards 0

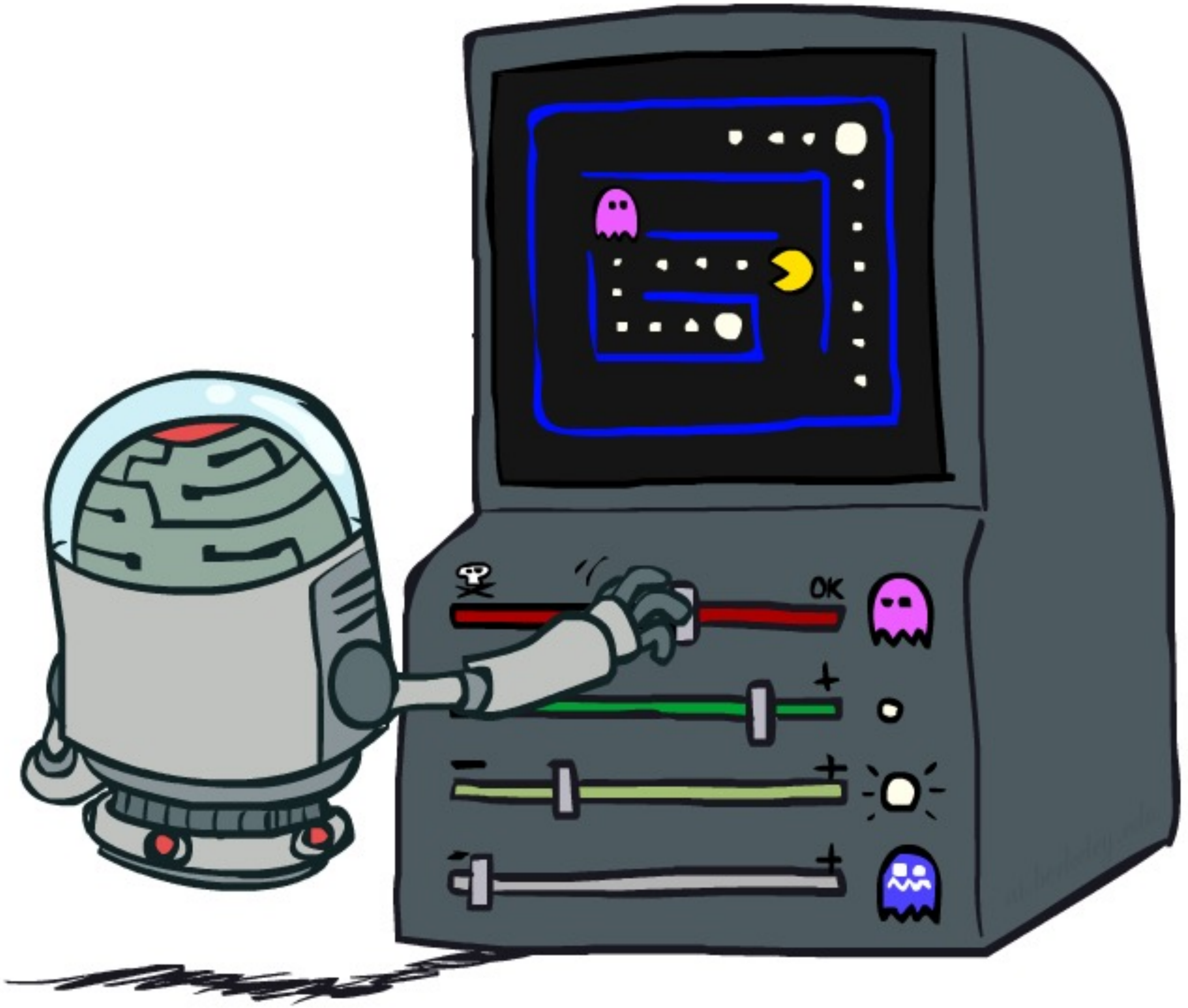
Demo Q-learning – Epsilon-Greedy – Crawler

Method 2: Optimistic Exploration Functions

- ***Exploration functions*** implement this tradeoff
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g., $f(u,n) = u + k/\sqrt{n}$
- Regular Q-update:
  - $Q(s,a) \leftarrow (1-\alpha) \cdot Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_a Q(s',a)]$
- Modified Q-update:
  - $Q(s,a) \leftarrow (1-\alpha) \cdot Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_a f(Q(s',a'),n(s',a'))]$
- Note: this propagates the "bonus" back to states that lead to unknown states as well!

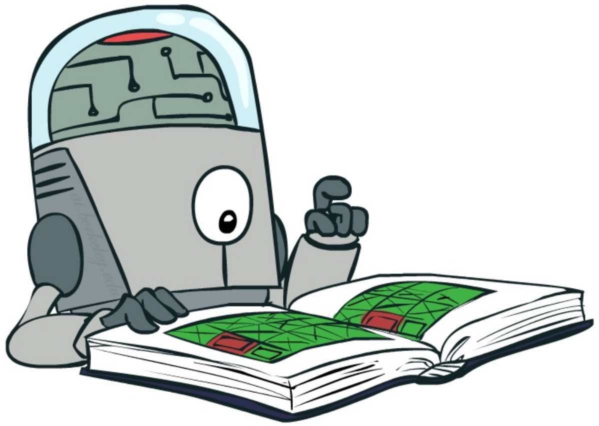
Demo Q-learning – Exploration Function – Crawler

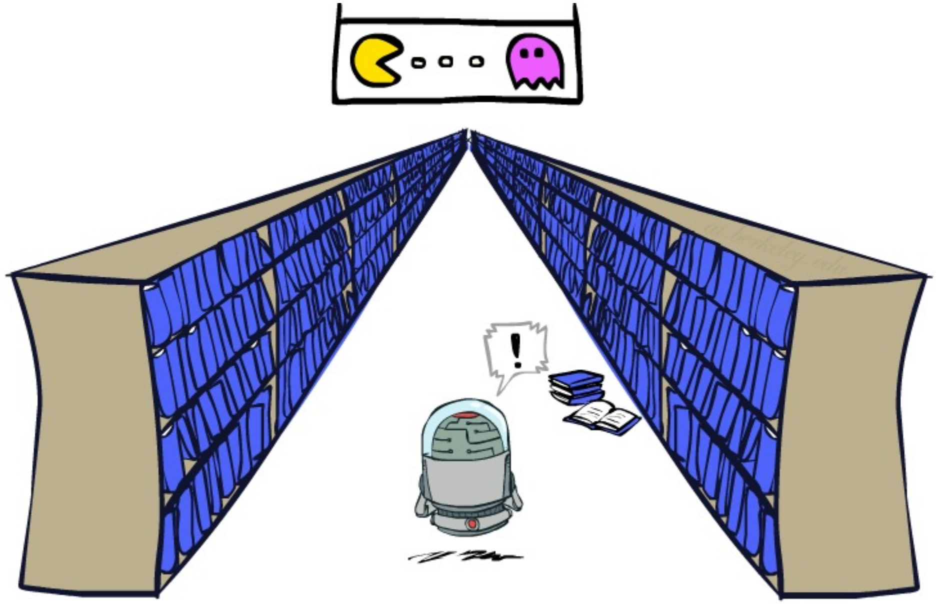
Approximate Q-Learning



Generalizing Across States

- Basic Q-Learning keeps a table of all Q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the Q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - Can we apply some machine learning tools to do this?

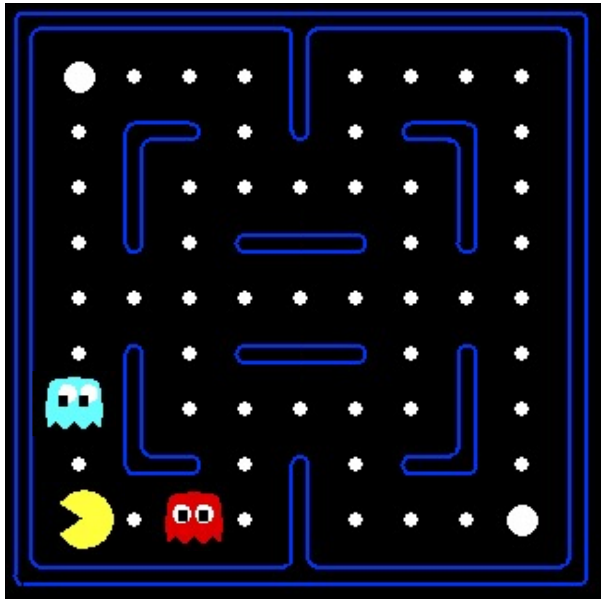




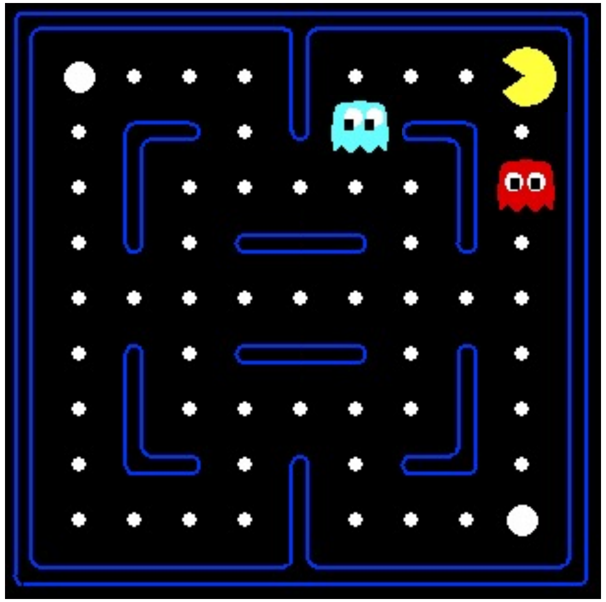
[demo – RL pacman]

Example: Pacman

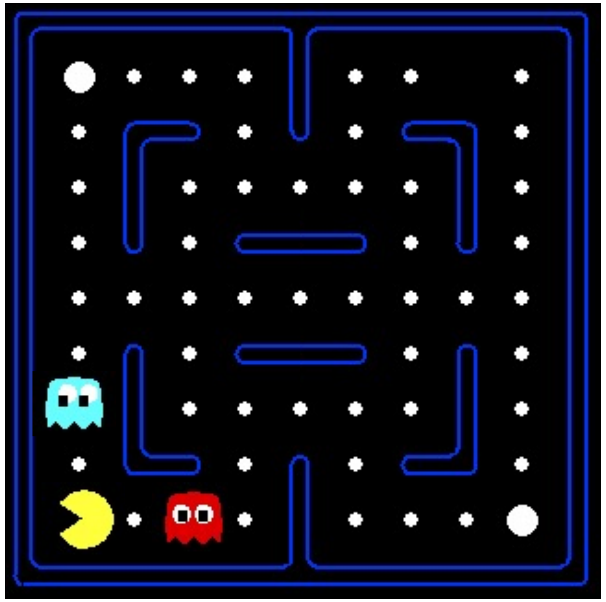
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:



Or even this one!



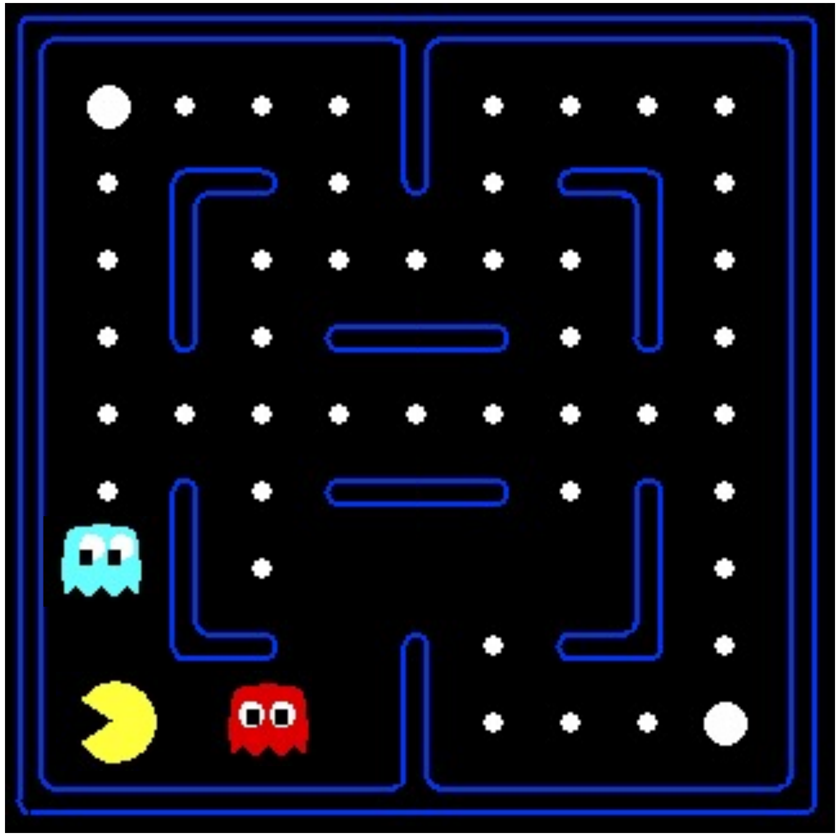
Demo Q-Learning Pacman – Tiny – Watch All

Demo Q-Learning Pacman – Tiny – Silent Train

Demo Q-Learning Pacman – Tricky – Watch All

Feature-Based Representations

- Solution: describe a state using a vector of ***features***
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost f_{GST}
    - Distance to closest dot
    - Number of ghosts
    - 1 / (distance to closest dot) f_{DOT}
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g., action moves closer to food)



Linear Value Functions

- We can express V and Q (approximately) as weighted linear functions of feature values:
  - $V_\theta(s) = \theta_1 f_1(s) + \theta_2 f_2(s) + \ldots + \theta_n f_n(s)$
  - $Q_\theta(s,a) = \theta_1 f_1(s,a) + \theta_2 f_2(s,a) + \ldots + \theta_n f_n(s,a)$

- Advantage: our experience is summed up in a few powerful numbers
  - Can compress a value function for chess (10^{43} states) down to about 30 weights!

- Disadvantage: states may share features but have very different expected utility!

SGD for Linear Value Functions

- Goal: Find parameter vector θ that minimizes the mean squared error between the true and approximate value function

$$J(\theta) = \mathbb{E}_\pi[\frac{1}{2}\big(V^\pi(s) - V_\theta(s)\big)^2]$$

- Stochastic gradient descent:

$$\begin{aligned} \theta &\leftarrow \theta - \alpha \frac{\partial J(\theta)}{\partial \theta} \\ &= \theta + \alpha\big(V^\pi(s) - V_\theta(s)\big)\frac{\partial V_\theta(s)}{\partial \theta} \end{aligned}$$

Supervised Learning for Value Function Approximation

- Let $V^{\pi}(s)$ denote the true target value function
- Use supervised learning on "training data" to predict the value function:

$$\langle s_1, G_1 \rangle, \langle s_2, G_2 \rangle, \dots, \langle s_T, G_T \rangle$$

- For each data sample

$$\theta \leftarrow \theta + \alpha \big(G_t - V_\theta(s) \big) x(s_t)$$

Temporal-Difference (TD) Learning Objective

$$\theta \leftarrow \theta + \alpha\big(V^{\pi}(s) - V_{\theta}(s)\big)x(s)$$

- In TD learning, $r_{t+1} + \gamma V_{\theta}(s_{t+1})$ is a data sample for the target
- Apply supervised learning on "training data":

$$\langle s_1, r_2 + \gamma V_{\theta}(s_2)\rangle, \langle s_2, r_3 + \gamma V_{\theta}(s_3)\rangle, \dots, \langle s_T, r_T\rangle$$

- For each data sample, update

$$\theta \leftarrow \theta + \alpha\big(r_{t+1} + \gamma V_{\theta}(s_{t+1}) - V_{\theta}(s)\big)x(s_t)$$

Q-Value Function Approximation

- Approximate the action-value function:

$$Q_\theta(s,a) \simeq Q^\pi(s,a)$$

- Objective: Minimize the **mean squared error**:

$$J(\theta) = \mathbb{E}_\pi\left[\frac{1}{2}\left(Q^\pi(s,a) - Q_\theta(s,a)\right)^2\right]$$

- Stochastic Gradient Descent on a single sample

$$\theta \leftarrow \theta + \alpha\Big(r_{t+1} + \gamma Q_\theta(s_{t+1}, a_{t+1}) - Q_\theta(s,a)\Big)\frac{\partial Q_\theta(s,a)}{\partial \theta}$$

Intuitive interpretation

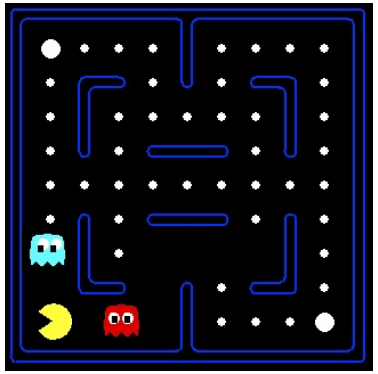
- Original Q-learning rule tries to reduce prediction error at s,a:
  - $Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$
- Instead, we update the weights to try to reduce the error at s,a:
  - $w_i \leftarrow w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] \, \partial Q_{\mathbf{w}}(s,a)/\partial w_i$

    $= w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] \, f_i(s,a)$
- Intuitive interpretation:
  - Adjust weights of active features
  - If something bad happens, blame the features we saw; decrease value of states with those features. If something good happens, increase value!

Example: Q-Pacman

$$Q(s,a) = 4.0\, f_{DOT}(s,a) - 1.0\, f_{GST}(s,a)$$

s

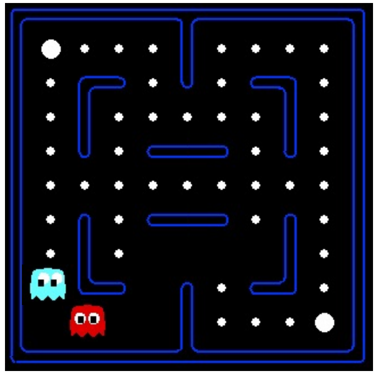


$f_{DOT}(s,NORTH) = 0.5$

$f_{GST}(s,NORTH) = 1.0$

a = NORTH
r = –500

s’



$Q(s,NORTH) = +1$

$r + \gamma \max_{a'} Q(s',a') = -500 + 0$

$Q(s',\cdot) = 0$

difference = –501

$w_{DOT} \leftarrow 4.0 + \alpha[-501]0.5$

$w_{GST} \leftarrow -1.0 + \alpha[-501]1.0$

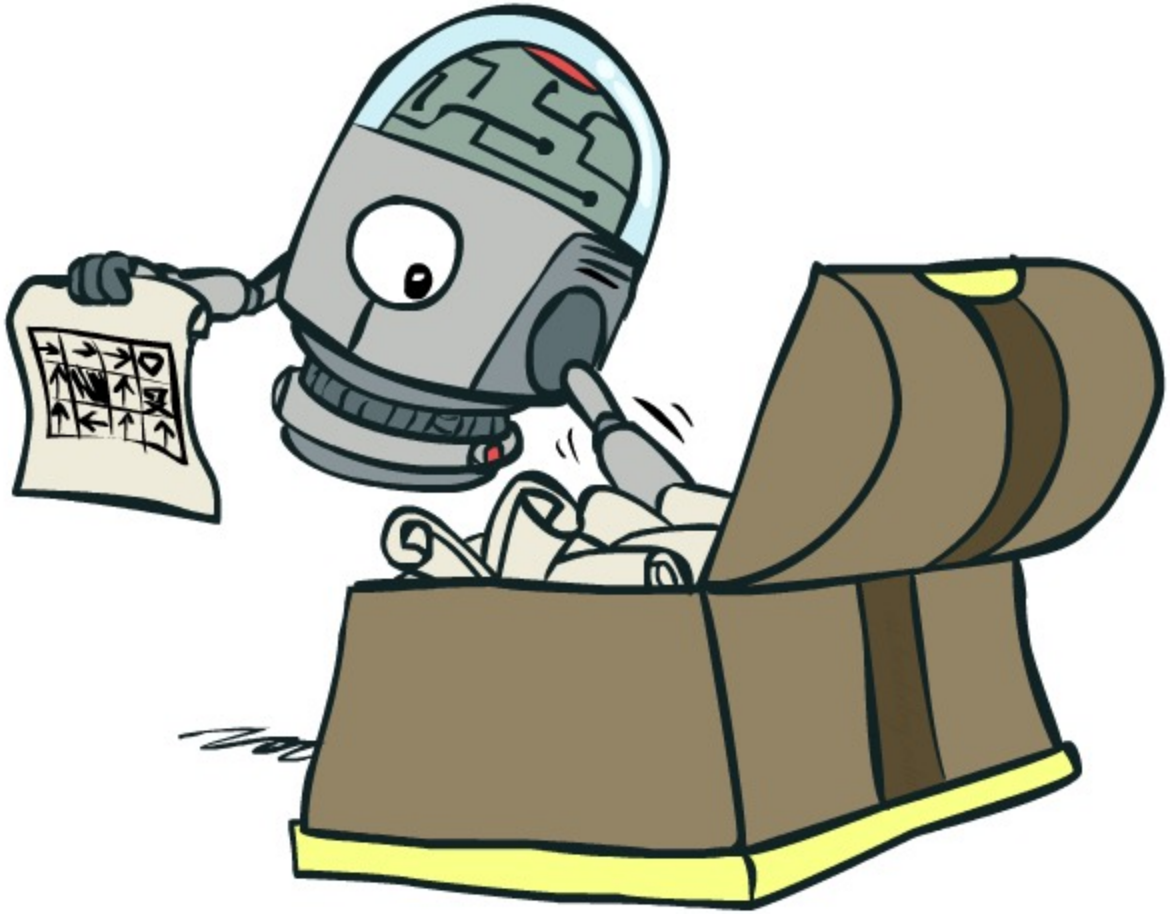
$$Q(s,a) = 3.0\, f_{DOT}(s,a) - 3.0\, f_{GST}(s,a)$$

Demo Approximate Q-Learning -- Pacman

Approaches to reinforcement learning

1. Model-based: Learn the model, solve it, execute the solution
2. Learn values from experiences, use to make decisions
    a. Direct evaluation
    b. Temporal difference learning
    c. Q-learning
3. Optimize the policy directly

Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)

- Solution: learn policies that maximize rewards, not the values that predict them

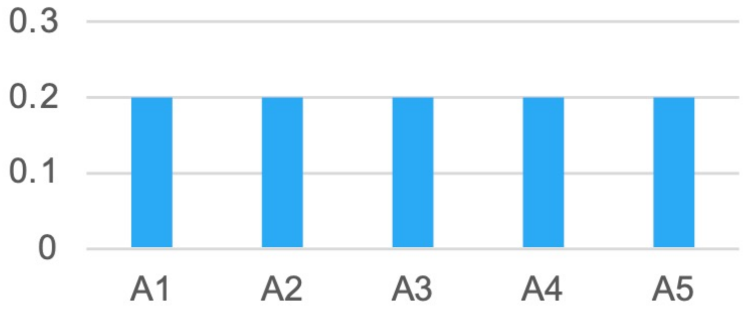
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by **hill climbing** (or gradient ascent!) on feature weights

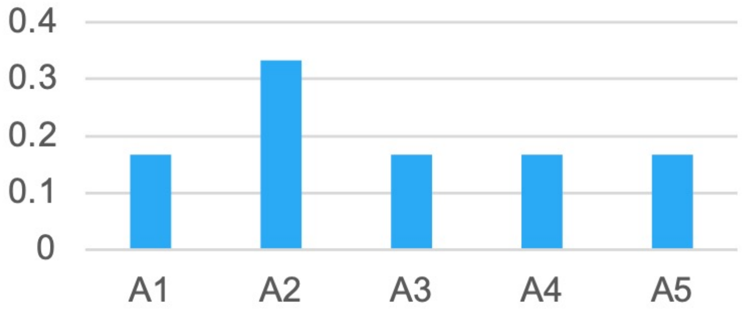
Parameterized Policy

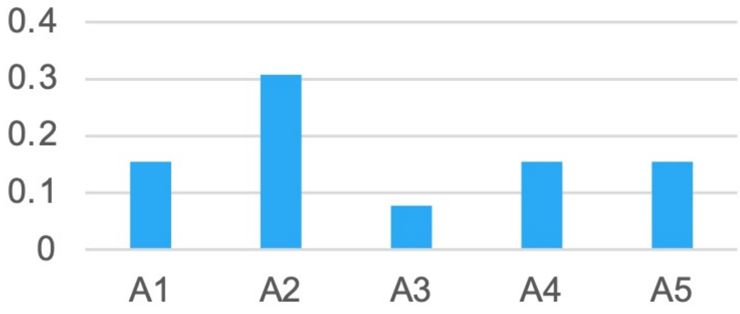
- A policy can be parameterized as $\pi_\theta(a|s)$
- The policy can be deterministic: $a = \pi_\theta(s)$
  - Or stochastic: $\pi_\theta(a|s) = P(a|s;\theta)$
- θ represents the parameters of the policy

Policy Gradient

- Simplest version:
  - Start with initial policy $\pi(s)$ that assigns probability to each action
  - Sample actions according to policy π
  - Update policy:
    - If an episode led to high utility, make sampled actions more likely
    - If an episode led to low utility, make sampled actions less likely







Policy Gradient in a Single-Step MDP

- Consider a simple single-step Markov Decision Process (MDP)
  - The initial state is drawn from a distribution: $s \sim d(s)$
  - The process terminates after one action, yielding a reward r_{sa}
- Expected Value of the Policy

$$J(\theta) = \mathbb{E}_{\pi_\theta}[r] = \sum_{s \in S} d(s) \sum_{a \in A} \pi_\theta(a|s) r_{sa}$$

$$\frac{\partial J(\theta)}{\partial \theta} = \sum_{s \in S} d(s) \sum_{a \in A} \frac{\partial \pi_\theta(a|s)}{\partial \theta} r_{sa}$$

Likelihood Ratio Trick

- Uses the identity:

$$\frac{\partial \pi_\theta(a|s)}{\partial \theta} = \pi_\theta(a|s)\frac{1}{\pi_\theta(a|s)}\frac{\partial \pi_\theta(a|s)}{\partial \theta}$$
$$= \pi_\theta(a|s)\frac{\partial \log \pi_\theta(a|s)}{\partial \theta}$$

- The gradient of the expected return can be written as:

$$J(\theta) = \mathbb{E}_{\pi_\theta}[r] = \sum_{s\in S} d(s) \sum_{a\in A} \pi_\theta(a|s)\boxed{r_{sa}}$$
$$\frac{\partial J(\theta)}{\partial \theta} = \sum_{s\in S} d(s) \sum_{a\in A} \frac{\partial \pi_\theta(a|s)}{\partial \theta} r_{sa}$$
$$= \sum_{s\in S} d(s) \sum_{a\in A} \pi_\theta(a|s)\boxed{\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} r_{sa}}$$
$$= \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} r_{sa}\right]$$

Can be approximated by sampling s from d(s) and a from π_θ

Extension to Multi-step MDP

- Replace the instantaneous reward r(s,a) with the Q-value

$$\frac{\partial J(\theta)}{\partial \theta} = \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} Q^{\pi_\theta}(s,a)\right]$$

REINFORCE Algorithm

- Use the cumulative reward G_t as an estimator for $Q^{\pi_\theta}(s,a)$

- initialize θ arbitrarily

  for each episode $\{s_1, a_1, r_2, \ldots, s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ do

      for $t = 1$ to $T - 1$ do

  $$\theta \leftarrow \theta + \alpha \frac{\partial}{\partial \theta} \log \pi_\theta(a_t | s_t) G_t$$

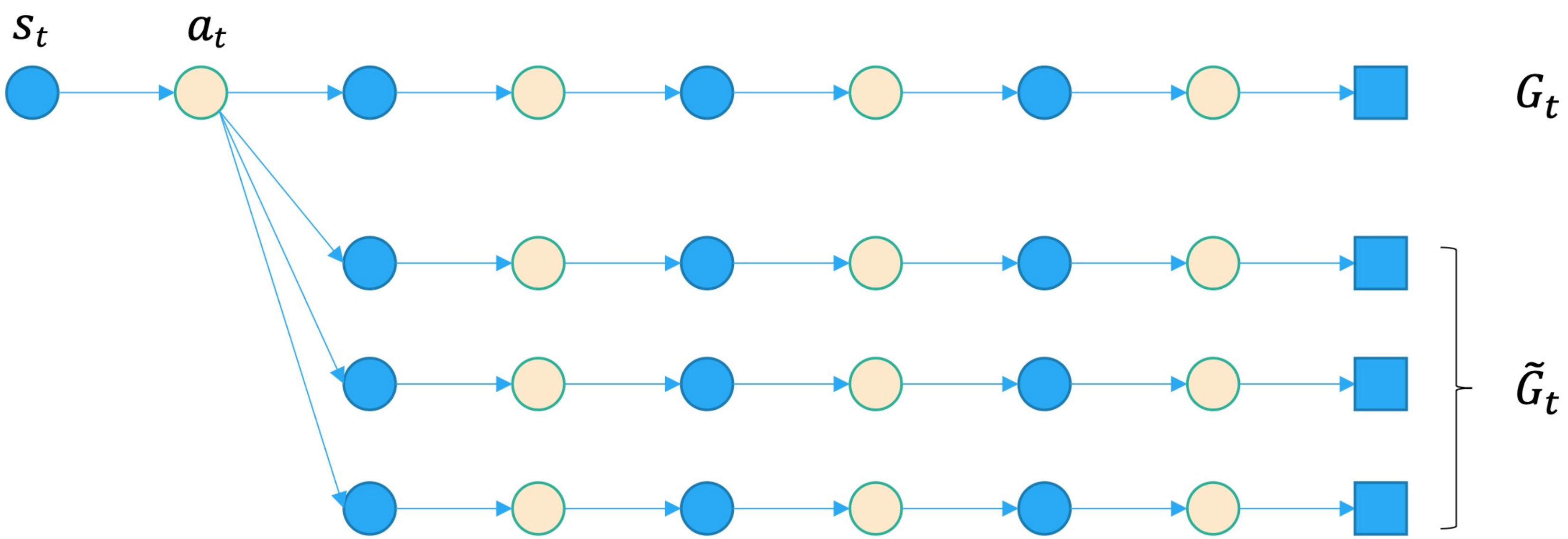
      end for

  end for

  return θ

REINFORCE Algorithm 2

- Can average multiple roll-out returns



$$\tilde{G}_t = \frac{1}{N}\sum_{i=1}^{n} G_t^{(i)}$$

Limitations of the REINFORCE Algorithm

- Episodic data requirement
  - REINFORCE typically requires tasks to terminate in order to compute the full return G_t
- Low data efficiency
  - In practice, REINFORCE needs a large amount of training data to achieve stable learning
- High variance in training (most critical issue)
  - The estimated returns from sampled trajectories can have very high variance, making gradient estimates noisy and unstable

Actor-Critic

- Intuition
  - REINFORCE estimates the policy gradient using Monte Carlo returns G_t to approximate $Q(s_t, a_t)$
  - Why not learn a trainable value function $Q_\phi(s, a)$ to estimate $Q^\pi(s, a)$ directly?
- Actor and critic

Actor $\pi_\theta(a|s)$

Improve the policy based on value estimates provided by the critic

Critic $Q_\Phi(s, a)$

Evaluate the value of actions taken by the actor's policy

Training of the Actor-Critic Algorithm

- Critic: $Q_\phi(s, a)$
  - Learns to accurately estimate the action-value under the current actor policy

$$Q_\Phi(s, a) \simeq r(s, a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a), a' \sim \pi_\theta(a'|s')}[Q_\Phi(s', a')]$$

- Actor: $\pi_\theta(a|s)$
  - Learns to take actions that maximize the critic's estimated value

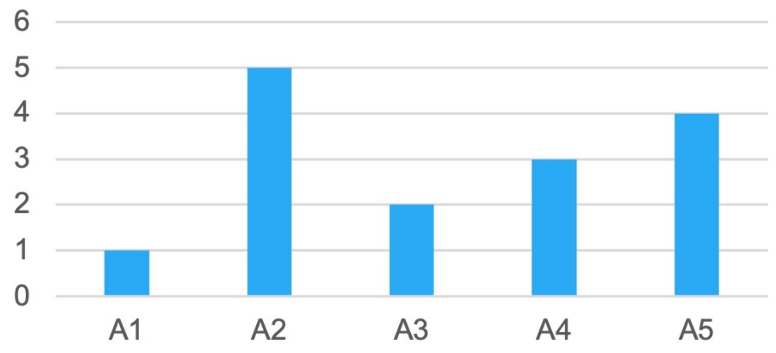
$$J(\theta) = \mathbb{E}_{s \sim p, \pi_\theta}[\pi_\theta(a|s) Q_\Phi(s, a)]$$

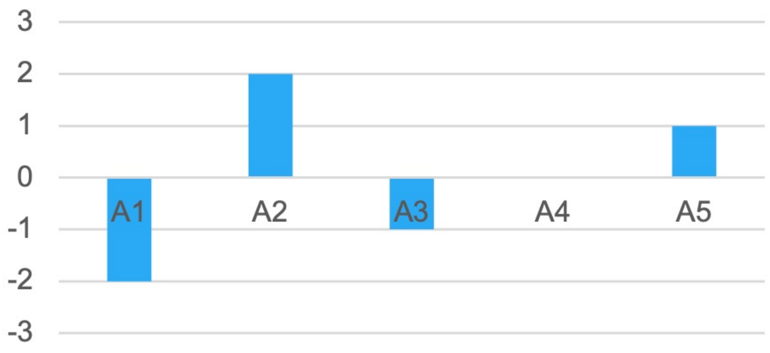
$$\frac{\partial J(\theta)}{\partial \theta} = \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} Q_\Phi(s, a)\right]$$

A2C: Advantageous Actor-Critic

- Idea: Normalize the critic's score by subtracting a baseline function (often a value function V(s))
  - Provides more informative feedback:
    - Decrease the probability of worse-than-average actions
    - Increase the probability of better-than-average actions
  - Helps to further reduce variance in policy gradient estimates

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$





Training of A2C

- Connection between Q-value and value function

$$
\begin{aligned}
Q^{\pi}(s,a) &= r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a), a' \sim \pi_\theta(a'|s')} [Q_\Phi(s',a')] \\
&= r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} [V^{\pi}(s')]
\end{aligned}
$$

- To approximate the advantage function

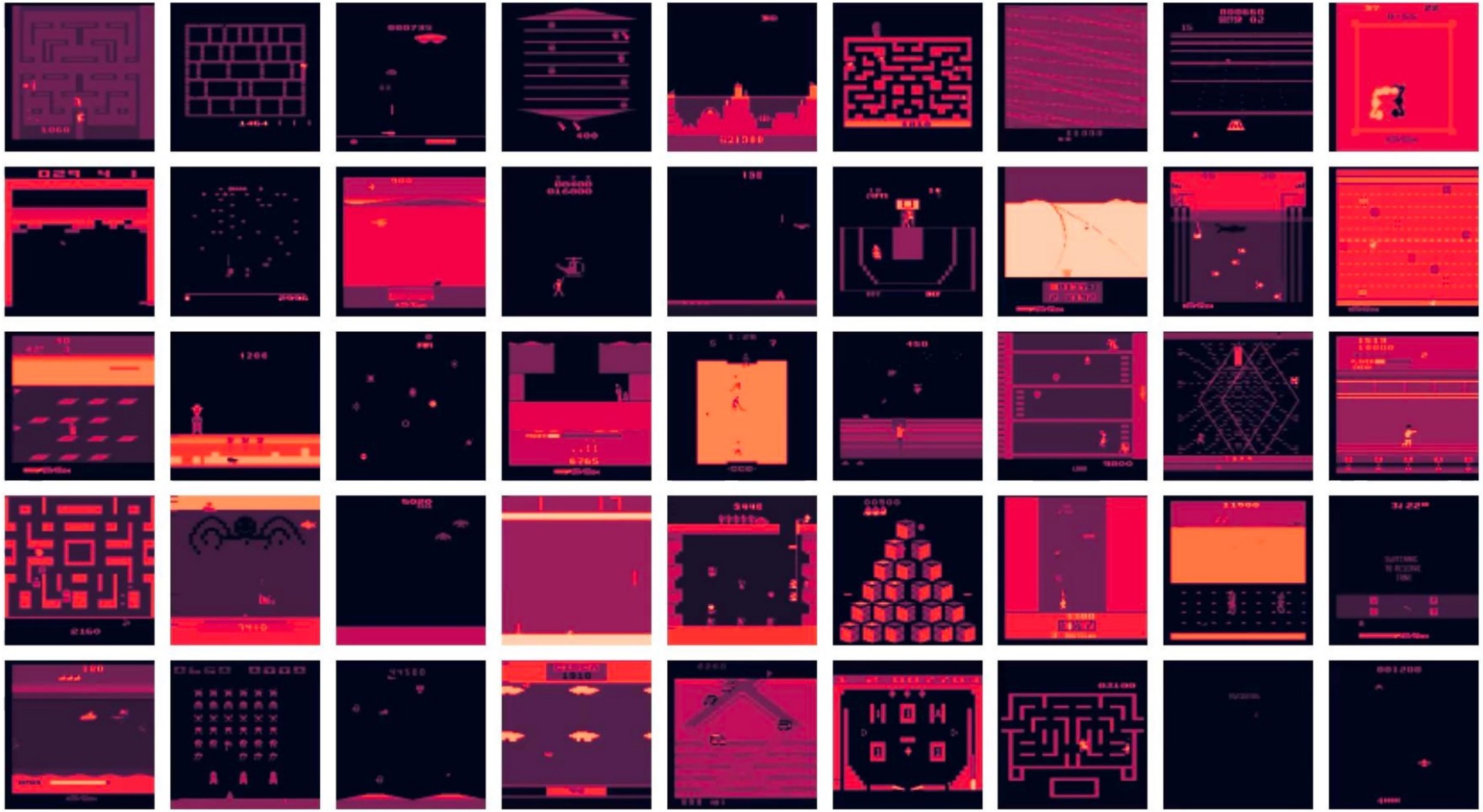
$$
\begin{aligned}
A^{\pi}(s,a) &= Q^{\pi}(s,a) - V^{\pi}(s) \\
&= r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} [V^{\pi}(s') - V^{\pi}(s)] \\
&\simeq r(s,a) + \gamma \left(V^{\pi}(s') - V^{\pi}(s) \right)
\end{aligned}
$$

Sample the next
state s‘

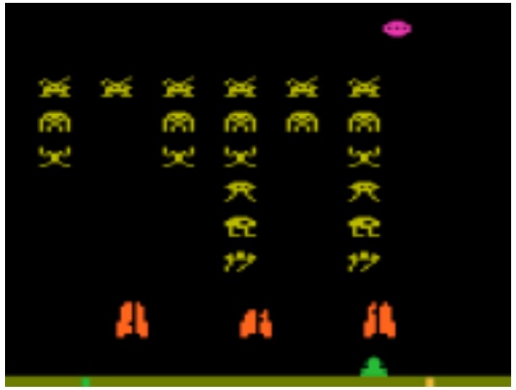
Case Studies of Reinforcement Learning!

- Atari game playing
- Robot Locomotion
- Language assistants

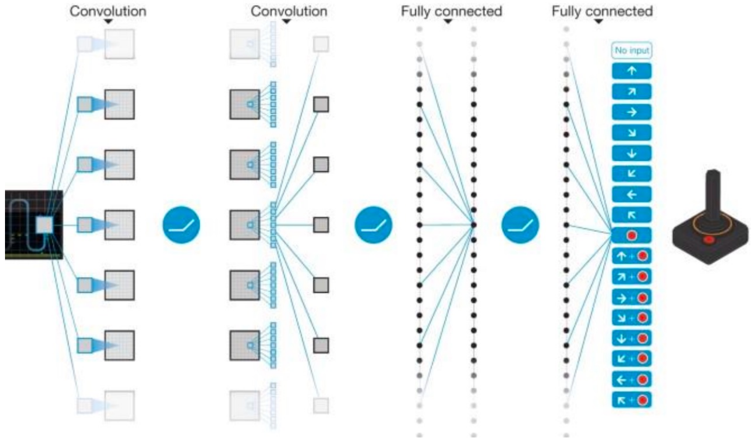
Case Studies: Atari Game Playing



Case Studies: Atari Game Playing



- MDP:
  - State: image of game screen
    - 25684*84 possible states
    - Processed with hand-designed feature vectors or neural networks
  - Action: combination of arrow keys + button (18)
  - Transition T: game code (don’t have access)
  - Reward R: game score (don’t have access)
- Very similar to our pacman MDP
- Use approximate Q learning with neural networks and ε-greedy exploration to solve



[Human-level control through deep reinforcement learning, Mnih et al, 2015]

Case Studies: Robot Locomotion

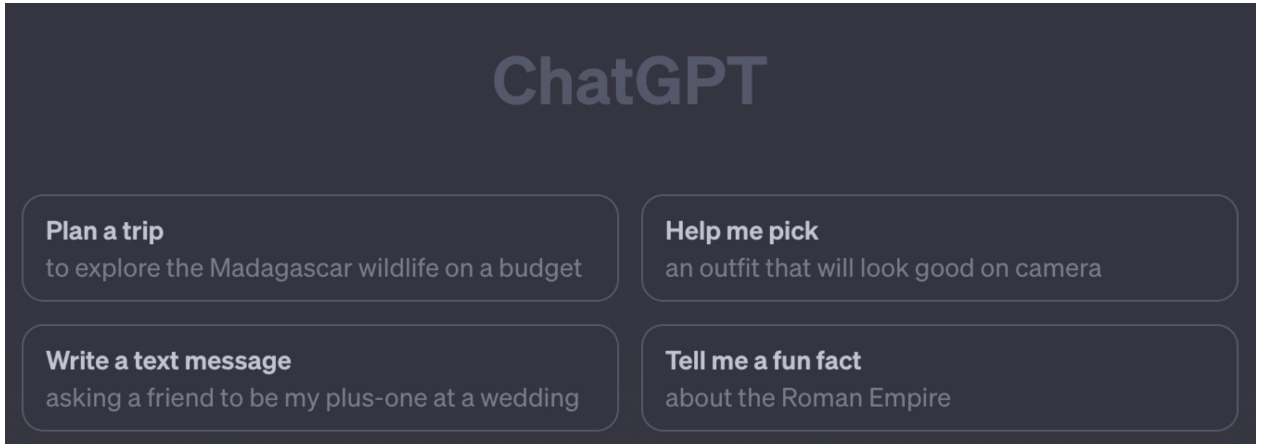
- https://www.youtube.com/watch?v=cqvAgcQl6s4

Case Studies: Robot Locomotion

- MDP:
  - State: image of robot camera + N joint angles + accelerometer + …
    - Angles are N-dimensional continuous vector!
    - Processed with hand-designed feature vectors or neural networks
  - Action: N motor commands (continuous vector!)
    - Can’t easily compute max $Q(s', a)$ when a is continuous
    - Use policy search methods or adapt Q learning to continuous actions
  - Transition T: real world (don’t have access)
  - Reward R: hand-designed rewards
    - Stay upright, keep forward velocity, etc
- Learning in the real world may be slow and unsafe
  - Build a simulator and learn there first, then deploy in real world

[Extreme Parkour with Legged Robots, Cheng et al, 2023]

Case Studies: Language Assistants



Case Studies: Language Assistants

- Step 1: train large language model to mimic human-written text
  - Query: “What is population of Berkeley?”
  - Human-like completion: “This question always fascinated me!”
- Step 2: fine-tune model to generate helpful text
  - Query: “What is population of Berkeley?”
  - Helpful completion: “It is 117,145 as of 2021 census”
- Use Reinforcement Learning in Step 2

Case Studies: Language Assistants

- MDP:
  - State: sequence of words seen so far (ex. “What is population of Berkeley? ”)
    - $100{,}000^{1{,}000}$ possible states
    - Huge, but can be processed with feature vectors or neural networks
  - Action: next word (ex. “It”, “chair”, “purple”, …) (so 100,000 actions)
    - Hard to compute max $Q(s', a)$ when max is over 100K actions!
  - Transition T: easy, just append action word to state words
    - s: “My name“ a: “is“ s’: “My name is“
  - Reward R: ???
    - Humans rate model completions (ex. “What is population of Berkeley? ”)
      - “It is 117,145“: +1 “It is 5“: -1 “Destroy all humans“: -1
    - Learn a reward model R and use that (model-based RL)
- Often use policy gradient (Proximal Policy Optimization) but looking into Q Learning

Summary

- Exploration in Q-learning
  - Epsilon greedy; optimistic function
- Scaling up with feature representations and approximation
- Policy gradient
  - REINFORCE; Actor-Critic
- Some case studies

- Next lecture: deep RL